An optimal transport problem with backward martingale constraints motivated by insider trading.

Dmitry Kramkov[*] Yan Xu[†]

October 28, 2020

Abstract

Given a probability measure ν on $\mathbb{R}^2$, we want to

$$\text{minimize} \quad \int c(x,y)d\gamma \quad \text{over} \quad \gamma \in \Gamma(\nu) \tag{1}$$

for the *covariance-type* cost function $c(x,y) = (y_1 - x_1)(y_2 - x_2)$, where $\Gamma(\nu)$ is the family of probability measure γ on $\mathbb{R}^2 \times \mathbb{R}^2$, that have ν as their y-marginal and make a martingale from the canonical two-dimensional process (x,y). Problem (1) belongs to the class of optimal transport problems with *backward* martingale constraints, in the sense that the initial x-marginal is part of the solution. The motivation comes from a version of Kyle's equilibrium with insider.

Our main result states that a probability measure $\gamma \in \Gamma(\nu)$ is optimal if and only if there is a maximal monotone set $G \subset \mathbb{R}^2$ such that (1) it supports the x-marginal of γ, and (2) $c(x,y) = \min_{z\in G} c(z,y)$ for every $(x,y) \in \operatorname{supp}\gamma$. Furthermore, if ν is continuous, then the solution is uniquely determined by the subdifferential of the concave function $u_G(y) = \inf_{z\in G}\{c(z,y) - y_1y_2\}$.

The presentation is based on a joint paper with Yan Xu available on ArXiv.

[*]Carnegie Mellon University, Department of Mathematical Sciences, 5000 Forbes Avenue, Pittsburgh, PA, 15213-3890, US. *Email:* kramkov@cmu.edu

[†]Carnegie Mellon University, Department of Mathematical Sciences, 5000 Forbes Avenue, Pittsburgh, PA, 15213-3890, US.. *Email:* yanx1@andrew.cmu.edu