

Moscow state Lomonosov University

Moscow School of Economics

Syllabus

"Mathematical methods for estimating the derivatives"

for Bachelors degree in Economics 080100

Authors:

Alexey N. Kurbatskiy, PhD in mathematics

Alexander V. Slovesnov, PhD in mathematics

Moscow

2013

Course description

This course is designed to introduce students to the basic ideas and methods valuing futures, options and other derivatives. This course provides students with experience in the methods and applications of financial mathematics to a wide range of theoretical and practical situations. "Mathematical methods for estimating the derivatives" is a two-semester course for the second and third year students studying at Moscow school of economics of Moscow state Lomonosov University.

This course is an important part of the bachelor stage in education of the future economists. It has give students skills for implementation of the mathematical knowledge and expertise to the problems of economics.

Prerequisites

Students are expected to have a basic knowledge in calculus, linear algebra, probability theory and mathematical statistics.

Course objectives

By the end of this course the students should:

- be acquired the students with the fundamentals of classical financial mathematics;
- know theoretical background and application boundaries of methods financial mathematics;
- get ready to solve practical problems concerning the valuing of the derivatives;
- develop their ability to apply their theoretical knowledge of financial mathematics which will enable them to model a written description of economic problems correctly and solve them by appropriate methods;
- develop their mathematical thinking.

Assessment

Teaching methods include lectures, discussion sections, practical work in computer class, homework, self-study.

The course is taught during 2 semesters. Total planned in-class load is 64 hours.

Control takes the following forms:

home assignments;

in the first semester, students sit a mid-term examination;

in the second semester, students have final examination.

The mark will be determined from the following activities:

- average grade for the home assignments (10%);
- first semester midterm exam (30%);
- final exam (60%).

Course outline

Section 1. Financial markets.

Operation of financial markets. Derivatives (forward and futures contracts, options). Types of traders (speculators, hedgers, arbitrageurs). Market participants. Specification of a futures contract. Profits and losses from futures contracts, dependency on the price of immediate delivery. Daily settlement and margins. Futures contracts on foreign currencies. Direct and indirect quotations. Bid and ask. Cross-rate calculation. Opening and closing out positions: types of orders. Basic principles of hedging. Types of hedgers. Hedging strategies using futures. Basis risk. Cross-hedging. Hedge ratio. Linear data dependency. Coefficient of correlation. Regression equation. Calculating the minimum variance hedge ratio. Types of interest rates: annual, semiannual, quarterly, daily compounding. Continuously compounded rate. Risk-free rates. The zero curve.

Section 2. Financial instruments.

Types of bonds. Cash flows. Present value of a cash flow. Bond pricing and bond yield. Sensitivity coefficient of bonds: duration, convexity. Determining treasury zero rates. Zero curve. Short selling mechanism. Forward price for an investment asset: with no income, with known income, with known yield. Comparison of forward prices and futures prices. Treasury bond futures. Day counts. Conversion factor. Cheapest-to-Delivery bond. Currency futures. Mechanics of interest rate swaps. Role of financial intermediary. Valuation of swaps. Types of options, put and call options. Specification of stock options, American and European options. Factors effecting option prices. Terms of trading: position and exercise limits, broker's commissions, ensuring short position. Stock splits. Indices of stock market. European stock options. Upper and lower bounds for European options prices. Put-call parity of European options. Early exercise of options on a non-dividend-paying stock. American stock options. Upper and lower bounds for American options prices. Analogue of put-call parity for American options prices. Effect of dividends. Trading strategies involving options. Strategies involving a single option and a stock. Spreads: types of spreads, payoffs. Combinations.

Section 3. Credit risk and credit derivatives.

The VaR measure. Historical simulation. Model building approach. Monte-Carlo simulation. Comparison of approaches. Linear and quadratic models for calculating VaR. Principal components analysis and his application for estimating VaR. Volatility. Estimating volatility. Standard estimate and weighting schemes.

The exponentially weighted moving average model. The GARCH(1,1) model. Maximum likelihood methods for estimating the parameters of the GARCH(1,1) model. Using GARCH(1,1) model to forecast future volatility of the stock and bond prices. Credit risk mitigation: netting, collateralization, downgrade triggers. A factor-based correlation structure. The binomial correlation measure. Credit VaR. Credit derivatives. Credit default swaps and credit indices. Valuation of credit default swaps, default probability estimates. Binary credit default swaps. Total return swaps. Basket credit default swaps and collateralized debt obligations. Valuation basket credit default swaps using Gaussian copula model.

Section 4. Mathematical methods for estimating financial instruments.

Binomial trees. A one step-binomial model for valuing European options. Probabilistic interpretation. A binomial tree for valuing American options. Using volatility for estimating value of an option. Valuing derivatives by Monte-Carlo simulation. Derivatives dependent on several market variables. Generating the random samples. Number of trials. Applications. Finite difference methods. Implicit and explicit finite difference methods. Change of variable. Relation to trinomial tree approaches.

Section 5. The Black-Scholes-Merton model.

The Markov property. Continuous-time stochastic processes. Wiener processes. Generalized Wiener processes. The process for a stock price. The Lognormal property. Ito process. Ito's lemma. Historical background. The Black-Scholes-Merton model. The ideas underlying the Black-Scholes-Merton model. The Black-Scholes-Merton differential equation. Application of the model to forward contracts. Black-Scholes formulas for the prices of a European option on a non-dividend-paying stock. Properties of Black-Scholes formulas. Implied volatilities. Volatility smiles. The reason of a smile in foreign currency options and in equity options. The volatility term structure. Volatility surfaces.

Class topical outline

№	Names of chapters and topics	Lectures	Practical exercises	Individual work ⁱ
	Chapter I. Financial markets.			
	Topic 1. Operation of financial markets. Derivatives. Types of traders.	2	0	2
	Topic 2. Futures market. Market participants. Specification of a futures contract. Profits and losses from futures contracts, dependency on the price of immediate delivery. Daily settlement and margins.	2	0	2
	Topic 3. Futures contracts on foreign currencies. Direct and indirect quotations. Bid and ask.	2	0	2
	Topic 4. Cross-rate calculation. Opening and closing out positions: types of orders. Basic principles of hedging. Types of hedgers. Hedging strategies using futures. Basis risk. Cross-hedging.	2	0	2
	Topic 5. Hedge ratio. Linear data dependency. Coefficient of correlation. Regression equation. Calculating the minimum variance hedge ratio.	0	2	2
	Topic 6. Types of interest rates: annual, semiannual, quarterly, daily compounding. Continuously compounded rate. Risk-free rates. The zero curve.	0	2	2
	<i>Sum-total for chapter I</i>	8	4	12
	Chapter II. Financial instruments.			
	Topic 1. Types of bonds. Cash flows. Present value of a cash flow. Bond pricing and bond yield.	2	0	2
	Topic 2. Sensitivity coefficient of bonds: duration, convexity. Determining treasury zero	1	1	2

	rates. Zero curve.			
	Topic 3. Short selling mechanism. Forward price for an investment asset: with no income, with known income, with known yield. Comparison of forward prices and futures prices.	2	0	2
	Topic 4. Treasury bond futures. Day counts. Conversion factor. Cheapest-to-Delivery bond. Currency futures.	2	0	2
	Topic 5. Mechanics of interest rate swaps. Role of financial intermediary. Valuation of swaps.	1	1	2
	Topic 6. Types of options, put and call options. Specification of stock options, American and European options. Factors effecting option prices. Terms of trading: position and exercise limits, broker's commissions, ensuring short position.	2	0	2
	Topic 7. Stock splits. Indices of stock market. European stock options. Upper and lower bounds for European options prices. Put-call parity of European options.	2	0	2
	Topic 8. Early exercise of options on a non-dividend-paying stock. American stock options. Upper and lower bounds for American options prices. Analogue of put-call parity for American options prices. Effect of dividends.	0	2	2
	Topic 9. Trading strategies involving options. Strategies involving a single option and a stock. Spreads: types of spreads, payoffs. Combinations.	0	2	2
	<i>Sum-total for chapter II.</i>	<i>12</i>	<i>6</i>	<i>18</i>
	<i>Sum-total for term I:</i>	<i>20</i>	<i>10</i>	<i>30</i>
	Chapter III. Credit risk and credit derivatives.			
	Topic 1. The VaR measure. Historical simulation. Model building approach. Monte-Carlo	1	1	2

	simulation. Comparison of approaches.			
	Topic 2. Linear and quadratic models for calculating VaR. Principal components analysis and his application for estimating VaR.	0	2	2
	Topic 3. Volatility. Estimating volatility. Standard estimate and weighting schemes. The exponentially weighted moving average model.	0	2	2
	Topic 4. The GARCH(1,1) model. Maximum likelihood methods for estimating the parameters of the GARCH(1,1) model. Using GARCH(1,1) model to forecast future volatility of the stock and bond prices.	1	1	2
	Topic 5. Default probability and intensity, степени возмещения. Valuing the probability of defaults depending on stock and bond prices. Credit risk mitigation: netting, collateralization, downgrade triggers.	2	0	2
	Topic 6. A factor-based correlation structure. The binomial correlation measure. Credit VaR.	2	0	2
	Topic 7. Credit derivatives. Credit default swaps and credit indices. Valuation of credit default swaps, default probability estimates. Binary credit default swaps.	1	1	2
	Topic 8. Total return swaps. Basket credit default swaps and collateralized debt obligations. Valuation basket credit default swaps using Gaussian copula model.	1	1	2
	<i>Sum-total for chapter III.</i>	<i>8</i>	<i>8</i>	<i>16</i>
	Chapter IV. Mathematical methods for estimating financial instruments.			
	Topic 1. Binomial trees. A one step-binomial model for valuing European options. Probabilistic interpretation.	2	0	2

	Topic 2. A binomial tree for valuing American options. Using volatility for estimating value of an option.	0	2	2
	Topic 3. Valuing derivatives by Monte-Carlo simulation. Derivatives dependent on several market variables. Generating the random samples. Number of trials. Applications.	2	0	2
	Topic 4. Finite difference methods. Implicit and explicit finite difference methods. Change of variable. Relation to trinomial tree approaches.	1	1	2
	<i>Sum-total for Chapter IV.</i>	5	3	8
	Chapter V. The Black-Scholes-Merton model.			
	Topic 1. The Markov property. Continuous-time stochastic processes. Wiener processes. Generalized Wiener processes.	2	0	2
	Topic 2. The process for a stock price. The Lognormal property. Ito process. Ito's lemma. Historical background.	2	0	2
	Topic 3. The Black-Scholes-Merton model. The ideas underlying the Black-Scholes-Merton model. The Black-Scholes-Merton differential equation. Application of the model to forward contracts.	1	1	2
	Topic 4. Black-Scholes formulas for the prices of a European option on a non-dividend-paying stock. Properties of Black-Scholes formulas. Implied volatilities.	0	2	2
	Topic 5. Volatility smiles. The reason of a smile in foreign currency options and in equity options. The volatility term structure. Volatility surfaces.	1	1	2
	<i>Sum-total for Chapter V.</i>	6	4	10
	Sum-total for term II:	19	15	34
	Sum-total for course:	39	25	64

Topical outline

Lecture 1. Introduction.

Operation of financial markets. Derivatives (forward and futures contracts, options). Types of traders (speculators, hedgers, arbitrageurs).

Lecture 2. Futures markets.

Market participants. Specification of a futures contract. Profits and losses from futures contracts, dependency on the price of immediate delivery. Daily settlement and margins.

Lecture 3. Foreign currency futures.

Futures contracts on foreign currencies. Direct and indirect quotations. Bid and ask.

Lecture 4. Hedging strategies.

Cross-rate calculation. Opening and closing out positions: types of orders. Basic principles of hedging. Types of hedgers. Hedging strategies using futures. Basis risk. Cross-hedging.

Lecture 5. Calculating the minimum variance hedge ratio.

Hedge ratio. Linear data dependency. Coefficient of correlation. Regression equation. Calculating the minimum variance hedge ratio.

Lecture 6. Interest rates.

Types of interest rates: annual, semiannual, quarterly, daily compounding. Continuously compounded rate. Risk-free rates. The zero curve.

Lecture 7. Bond market.

Types of bonds. Cash flows. Present value of a cash flow. Bond pricing and bond yield.

Lecture 8. Bond characteristics.

Sensitivity coefficient of bonds: duration, convexity. Determining treasury zero rates. Zero curve.

Lecture 9. Forward and futures prices.

Short selling mechanism. Forward price for an investment asset: with no income, with known income, with known yield. Comparison of forward prices and futures prices.

Lecture 10. Bond futures.

Treasury bond futures. Day counts. Conversion factor. Cheapest-to-Delivery bond. Currency futures.

Lecture 11. Swaps.

Mechanics of interest rate swaps. Role of financial intermediary. Valuation of swaps.

Lecture 12. Option markets.

Types of options, put and call options. Specification of stock options, American and European options. Factors effecting option prices. Terms of trading: position and exercise limits, broker's commissions, ensuring short position.

Lecture 13. Stock markets.

Stock splits. Indices of stock market. European stock options. Upper and lower bounds for European options prices. Put-call parity of European options.

Lecture 14. Dividends.

Early exercise of options on a non-dividend-paying stock. American stock options. Upper and lower bounds for American options prices. Analogue of put-call parity for American options prices. Effect of dividends.

Lecture 15. Trading strategies.

Trading strategies involving options. Strategies involving a single option and a stock. Spreads: types of spreads, payoffs. Combinations.

Lecture 16. Value at risk.

The VaR measure. Historical simulation. Model building approach. Monte-Carlo simulation. Comparison of approaches.

Lecture 17. Calculating VaR.

Linear and quadratic models for calculating VaR. Principal components analysis and his application for estimating VaR.

Lecture 18. Estimating volatility.

Volatility. Estimating volatility. Standard estimate and weighting schemes. The exponentially weighted moving average model.

Lecture 19. GARCH(1,1) model.

The GARCH(1,1) model. Maximum likelihood methods for estimating the parameters of the GARCH(1,1) model. Using GARCH(1,1) model to forecast future volatility of the stock and bond prices.

Lecture 20. Credit risk.

Default probability and intensity, степени возмещения. Valuing the probability of defaults depending on stock and bond prices. Credit risk mitigation: netting, collateralization, downgrade triggers.

Lecture 21. Default correlation.

A factor-based correlation structure. The binomial correlation measure. Credit VaR.

Lecture 22. Credit default swaps.

Credit derivatives. Credit default swaps and credit indices. Valuation of credit default swaps, default probability estimates. Binary credit default swaps.

Lecture 23. Basket swaps.

Total return swaps. Basket credit default swaps and collateralized debt obligations. Valuation basket credit default swaps using Gaussian copula model.

Lecture 24. Binomial trees.

Binomial trees. A one step-binomial model for valuing European options. Probabilistic interpretation.

Lecture 25. Two step-binomial tree.

A binomial tree for valuing American options. Using volatility for estimating value of an option.

Lecture 26. Monte-Carlo simulation.

Valuing derivatives by Monte-Carlo simulation. Derivatives dependent on several market variables. Generating the random samples. Number of trials. Applications.

Lecture 27. Finite difference method.

Finite difference methods. Implicit and explicit finite difference methods. Change of variable. Relation to trinomial tree approaches.

Lecture 28. Wiener processes.

The Markov property. Continuous-time stochastic processes. Wiener processes. Generalized Wiener processes.

Lecture 29. Ito's lemma.

The process for a stock price. The Lognormal property. Ito process. Ito's lemma. Historical background.

Lecture 30. The Black-Scholes-Merton model.

The Black-Scholes-Merton model. The ideas underlying the Black-Scholes-Merton model. The Black-Scholes-Merton differential equation. Application of the model to forward contracts.

Lecture 31. Using Black-Scholes formulas for estimating the prices of a European option.

Black-Scholes formulas for the prices of a European option on a non-dividend-paying stock. Properties of Black-Scholes formulas. Implied volatilities.

Lecture 32. Applications of the Black-Scholes model.

Volatility smiles. The reason of a smile in foreign currency options and in equity options. The volatility term structure. Volatility surfaces.

Main reading

1. John C. Hull. Options, futures and other derivatives. Pearson Prentice Hall, 2009.
2. Fabozzi F.J., Rachev S.T., Focardi S.M. Financial Econometrics: From Basics to Advanced Modeling Techniques. Wiley, 2007.

Supplementary reading

3. Terence C. Mills, Raphael N. Markellos. The econometric modelling of financial time series, Cambridge University Press, 2008.
4. Joulenev S. V., Introduction to classical financial mathematics, Moscow University Press (in Russian).
5. A. N. Shiryaev, (1999). Essentials of stochastic finance. Facts, models, theory, Advanced Series on Statistical Science and Applied Probability, 3, World Scientific Publishing Co. Inc., River Edge, NJ, ISBN:981-02-3605-0, xvi+834 pp.
6. Shiryaev A.N. (1995). Probability. 2nd ed. New York: Springer-Verlag.

ⁱ Voluntary.